

Monte Carlo simulations of the three-dimensional XY spin glass focusing on the chiral and the spin order

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The ordering of the three-dimensional isotropic XY spin glass with the nearest-neighbor random Gaussian coupling is studied by extensive Monte Carlo simulations. To investigate the ordering of the spin and the chirality, we compute several independent physical quantities including the glass order parameter, the Binder parameter, the correlation-length ratio, the overlap distribution and the non-self-averageness parameter, *etc.*, for both the spin-glass (SG) and the chiral-glass (CG) degrees of freedom. Evidence of the spin-chirality decoupling, *i.e.*, the CG and the SG order occurring at two separated temperatures, $T_{CG} > T_{SG} > 0$, is obtained from the glass order parameter, which is fully corroborated by the Binder parameter. By contrast, the CG correlation-length ratio yields a rather pathological and inconsistent result in the range of sizes we studied, which may originate from the finite-size effect associated with a significant deviation of the spatial CG correlations from the standard Ornstein-Zernike form. Finite-size-scaling analysis yields the CG exponents $\nu_{CG} = 1.4 \pm 0.1$ and $\eta_{CG} = 0.29 \pm 0.12$, and the SG exponents $\nu_{SG} = 1.23^{+0.17}_{-0.06}$ and $\eta_{SG} = -0.42^{+0.12}_{-0.27}$. The obtained exponents are close to those of the Heisenberg SG, but are largely different from those of the Ising SG. The chiral overlap distribution and the chiral Binder parameter exhibit the feature of a continuous one-step replica-symmetry breaking (1RSB), consistently with the previous reports. Such a 1RSB feature is again in common with that of the Heisenberg SG, but is different from the Ising one, which may be the cause of the difference in the CG critical properties from the Ising SG ones despite of a common Z_2 symmetry.

I. INTRODUCTION

In spite of long history of research, spin glass (SG) is still a hot topic in statistical physics¹. It is a typical system possessing both strong frustration and randomness, leading to several extraordinary behaviors such as the slow dynamics and the rejuvenation-memory effect. In a theoretical treatment of SG, Edwards and Anderson (EA) proposed as early as in 1972 a simple model², the so-called EA model, in which the spins are put on each site of a regular lattice and interact via the random coupling taking both positive and negative signs. The infinite-range or the mean-field version of the EA model, first presented by Sherrington and Kirkpatrick (SK)³, was solved by Parisi revealing an intriguing concept of the replica symmetry breaking (RSB)⁴. For both cases of the Ising and the Heisenberg SK models, the relevant RSB turned out to be of hierarchical nature. In spite of such success of the mean-field theory, understanding the nature of the ordering of the finite-range EA model in three dimensions (3D) still remains incomplete. Numerical simulations have been the main tool in attacking the issue. Although the existence of a finite-temperature SG transition was established in the 3D Ising EA model^{5–11}, earlier numerical simulations on the 3D XY and the Heisenberg EA models suggested the absence of a finite-temperature transition^{12–18}, in apparent contrast to experiments^{19–26}.

Some time ago, in discussing the ordering of frustrated vector spin systems, Villain proposed a possible significance of the “chirality” degree of freedom, an Ising-like scalar quantity which represents the handedness of the

noncollinear spin structure. Villain made a conjecture that the 3D XY SG might exhibit a finite-temperature SG ordering by noting the Ising nature of the chirality and by invoking the occurrence of a finite-temperature SG transition in the 3D Ising SG²⁷.

Numerical simulations on the vector SG, *i.e.*, the two-component XY SG or the three-component Heisenberg SG, investigating both the spin and the chirality degrees of freedom, have been performed ever since 1985²⁸. A crucially important concept which emerged from these studies is the possible “spin-chirality decoupling” phenomenon^{29–31}. In 3D, it means that the chirality orders at a temperature higher than the spin, with an intermediate “chiral-glass” (CG) phase where only the chirality exhibits a glassy long-range order while the standard SG order still remains short-ranged. In dimensions lower than three, where both the spin and the chirality order only at $T = 0$, the spin-chirality decoupling means that the spin and the chiral correlation-length exponents are mutually different, *i.e.*, the existence of two different diverging length scales at the $T = 0$ transition^{32–45}. The concept of the “spin-chirality decoupling” in SG was originally proposed by one of the present authors (H.K.), and subsequently explored further by the corroborators for a wide range of systems. In terms of a symmetry, in the CG phase the Z_2 spin-reflection symmetry is spontaneously broken with keeping the $SO(3)$ or $SO(2)$ spin-rotation symmetry unbroken.

In case of the 3D XY SG, which is a target of the present study, the spin-chirality decoupling was first examined by the numerical domain-wall renormalization-group calculation and also by a Monte Carlo (MC) sim-

ulation^{35,46,47}. Further interesting features revealed by these numerical analyses might be the possible one-step RSB (1RSB)-like feature of the CG ordered state, and the non-Ising character of the CG criticality⁴⁷.

Similar spin-chirality decoupling phenomena, the 1RSB-like nature of the CG ordered state and the non-Ising character of the CG criticality were also observed in the 3D Heisenberg SG, a reference model of many of realistic SG materials including canonical SG, in spite of the difference in the nature of the chiralities relevant to the XY and the Heisenberg spins. For the former, the chirality is quadratic in spins and time-reversal even, while, in the latter, it is cubic in spins and time-reversal odd. Indeed, on the basis of the spin-chirality decoupling picture of the 3D Heisenberg SG, the chirality scenario of the experimental SG order was advanced²⁹⁻³¹. Recent large-scale simulations have revealed that the SG order actually takes place at a nonzero temperature⁴⁸⁻⁶¹, in contrast to earlier beliefs. Besides, most of recent simulations point to the occurrence of the spin-chirality decoupling in the system⁴⁸⁻⁵⁷, *i.e.*, $0 < T_{SG} < T_{CG}$. Several experimental facts were also successfully explained by the chirality scenario^{26,31}, and it becomes increasingly clear now that the spin-chirality decoupling is an indispensable concept in understanding the realistic SG systems.

On the other hand, the situation in the 3D XY SG seems less clear. Extensive calculations comparable in their scale to those of the 3D Heisenberg SG have been scarce, and the occurrence of the spin-chirality decoupling still remains controversial. Maucourt and Grempe suggested on the basis of their $T = 0$ domain-wall renormalization-group calculation for lattices $L \leq 8$ the occurrence of a nonzero T_{SG} located below T_{CG} ⁶². Mentioning some of the recent MC simulations on the model, Kawamura and Li simulated the $\pm J$ EA model by an equilibrium MC simulation up to the linear size $L = 16$, suggesting the occurrence of the spin-chirality decoupling⁴⁷, whereas Granato performed the dynamical Langevin simulations of the model for lattices of $L \leq 12$, to conclude the occurrence of a single transition $T_{SG} = T_{CG}$ ^{63,64}. Nakamura and collaborators performed the nonequilibrium relaxation analysis for lattices up to $L = 55$ (this method enables one to treat relatively larger sizes but some drawbacks appear in its short-time observations), and suggested that T_{SG} and T_{CG} were identical or close even if they were to be different^{65,66}. Young and corroborators investigated the Gaussian EA model by equilibrium MC simulations for lattices up to $L \leq 24$, reporting no evidence of the spin-chirality decoupling^{67,68}.

This confusing situation motivates us to re-examine the ordering of the 3D XY EA model with the random Gaussian coupling by large-scale MC simulations by treating large sizes up to $L = 40$, considerably larger than the sizes studied before by equilibrium simulations. A large number of samples of order $N_s \sim O(10^3)$ are simulated to obtain reasonable statistics. Furthermore, we compute various independent quantities including the glass order parameter, the Binder parameter, the corre-

lation length ratio, the overlap distribution function and the non-self-averageness parameters both for the SG and the CG, in order to check consistency among various independent quantities.

We note that the 3D XY EA model is a reference model for SG magnets with an easy-plane-type uniaxial magnetic anisotropy⁶⁹⁻⁷⁴. Readers are referred to Ref. 74 for detailed discussion. The ordering properties of the model would also be helpful in understanding the peculiar ordering behaviors experimentally observed in these granular cuprate superconductors⁷⁵⁻⁸².

Overall, the results of our large-scale simulations speak for the occurrence of the spin-chirality decoupling in the 3D XY SG. The estimated SG and CG transition temperatures are $T_s = 0.274^{+0.012}_{-0.032}$ and $T_{CG} = 0.308 \pm 0.05$, T_{CG} being higher than T_{SG} by about 10%. In estimating the transition temperatures, we have found a reasonably good consistency among various independent quantities, with one exception of the CG correlation-length ratio, which behaves rather badly leading to a pathological estimate of T_{CG} . Thus, in deriving the above estimate of T_{CG} , we have not used the CG-correlation length data, in contrast to Ref. 67. To clarify the origin of the observed pathological behavior of the CG correlation length, we directly compute the spatial chiral correlation function to find that the standard Ornstein-Zernike (OZ) form may be inappropriate to describe the CG correlation length in the range of small sizes in which we make our simulations.

The critical properties associated with the SG and the CG orderings are also examined. We obtain the CG exponents $\nu_{CG} = 1.4 \pm 0.1$ and $\eta_{CG} = 0.29 \pm 0.12$, while the SG ones $\nu_{SG} = 1.23^{+0.17}_{-0.06}$ and $\eta_{SG} = -0.42^{+0.12}_{-0.27}$, where ν and η are the correlation-length and the critical-point-decay exponents, respectively. These exponents turn out to be close to the corresponding Heisenberg SG exponents, but are different from the Ising SG ones. We also confirm the 1RSB nature of the ordered state, consistently with the previous reports.

This paper is organized as follows. In Sec. II, we introduce the model and explain some of the details of our simulation. In Sec. III, we define several physical quantities which we compute to examine the SG and the CG orderings. The results of our MC simulations are presented in Sec. IV. We show the data of the glass order parameter, the Binder parameter, the correlation length ratio, the overlap distribution function, the non-self-averageness parameters and the spatial correlation function for both the SG and the CG degrees of freedom. The CG and the SG transition temperatures are estimated via an infinite-size extrapolation of appropriate finite-size data. Possible RSB character of the ordered state is examined in this section. In Sec. V, the critical properties of the SG and the CG transitions are analyzed, and the CG and the SG critical exponents are determined. Comparison is made with the corresponding exponents of the 3D Heisenberg and of the 3D Ising SGs. The last section is devoted to summary and discussion. In appendix, the behavior of the SG Binder parameter

in the thermodynamic limit across the CG and the SG transition points is analyzed.

II. MODEL AND SIMULATIONS

The model we study is the isotropic XY EA model on a 3D simple-cubic lattice. The sites are labeled by the index i ($i = 1, 2, \dots, N$), the corresponding coordinate being denoted as $\mathbf{r}_i = (x_i, y_i, z_i)$. The total number of spins N is related to the linear system size L as $N = L^3$. The XY spin on the i th site, \mathbf{S}_i , has two components, $\mathbf{S}_i = (S_{ix}, S_{iy}) = (\cos \theta_i, \sin \theta_i)$ where $0 \leq \theta_i < 2\pi$. The Hamiltonian is given by

$$\mathcal{H} = - \sum_{\langle i, j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \quad (1)$$

where the summation $\langle i, j \rangle$ is taken over the all nearest-neighbor pairs. The interaction J_{ij} is a random Gaussian variable whose mean and variance are taken to be zero and unity, respectively. The partition function is given by

$$Z = \int \prod_{i=1}^N \frac{d\theta_i}{2\pi} e^{-\beta \mathcal{H}}, \quad (2)$$

where β is the inverse temperature $1/T$ normalized by the Boltzmann constant k_B . The thermal average will be denoted by the angular brackets $\langle \dots \rangle$.

We perform MC simulations based on the single-spin-flip Metropolis method combined with the over-relaxation method and the temperature-exchange technique. This algorithm is known to effectively reduce the long correlation time involved in simulation of hard-relaxing systems such as SG.

In a unit process of the over-relaxation, we compute first the local field felt by the spin at site i , $\mathbf{h}_i = \sum_{j \in \Lambda_i} J_{ij} \mathbf{S}_j$ where Λ_i represents the neighbors of the site i , and then reflect the spin \mathbf{S}_i with respect to the local field \mathbf{h}_i as

$$\mathbf{S}_i \rightarrow \mathbf{S}'_i = -\mathbf{S}_i + 2 \frac{\mathbf{S}_i \cdot \mathbf{h}_i}{h_i^2} \mathbf{h}_i. \quad (3)$$

The simple cubic lattice consists of two interpenetrating sublattices, and we perform the Metropolis update sequentially through the sites on one sublattice after another, which is followed by the M -times over-relaxation sweeps, also performed sequentially through the sites on each sublattice. This procedure consists our unit MC step. In our simulations, we take M equal to the linear system size L .

In the temperature-exchange process, we prepare N_T spin configurations at a set of temperatures distributed between T_{\min} and T_{\max} . The maximum temperature T_{\max} is chosen to be high enough where the autocorrelation times of the spin and the chirality are sufficiently short even in the single-spin-flip dynamics, typically 40

MC steps per spin (MCS). Each trial of the temperature exchange is performed for a pair of neighboring temperatures. A temperature-exchange trial is done after every MCS.

In Table I, we summarize the simulation parameters employed in our MC simulations, which include the linear system size L , the total number of samples averaged (independent bond realizations) N_s , the number of MCS discarded for equilibration N_{MC1} , the number of MCS employed for measuring physical quantities N_{MC2} , the maximum and the minimum temperatures in the temperature-exchange process T_{\max} and T_{\min} , and the number of temperature points N_T . In simulating SG systems, special care has to be taken for equilibration. In this paper, we follow the criteria of Ref. 55. Error bars are estimated by using the bootstrap method from sample to sample fluctuations.

| L | N_s | N_{MC1} | N_{MC2} | T_{\max} | T_{\min} | N_T |
|-----|-------|---------------------|---------------------|------------|------------|-------|
| 4 | 5000 | 1×10^4 | 1×10^5 | 0.86 | 0.24 | 32 |
| 6 | 5000 | 3×10^4 | 1×10^5 | 0.86 | 0.24 | 32 |
| 8 | 2000 | 5×10^4 | 1×10^5 | 0.80 | 0.24 | 32 |
| 12 | 2000 | 1×10^5 | 1×10^5 | 0.60 | 0.24 | 32 |
| 16 | 2048 | 4×10^5 | 4×10^5 | 0.52 | 0.26 | 32 |
| 20 | 1024 | 5×10^5 | 5×10^5 | 0.50 | 0.266 | 40 |
| 24 | 1024 | 7.5×10^5 | 7.5×10^5 | 0.49 | 0.271 | 40 |
| 32 | 1024 | 1.5×10^6 | 1.5×10^6 | 0.48 | 0.2736 | 56 |
| 40 | 384 | $2.2.8 \times 10^6$ | $2.2.8 \times 10^6$ | 0.46 | 0.2891 | 64 |
| | 128 | $2.3.4 \times 10^6$ | $2.3.4 \times 10^6$ | 0.442 | 0.2792 | 64 |

TABLE I. Parameters of our Monte Carlo simulations. L is the linear system size, N_s is the total number of samples, N_{MC1} is the Monte Carlo steps per spin discarded for equilibration, N_{MC2} is the Monte Carlo steps per spin subsequently used in measuring physical quantities, T_{\max} and T_{\min} are the highest and the lowest temperatures employed in the temperature-exchange process, and N_T is the total number of temperature points. For $L = 40$, we use two different temperature sets, where N_{MC1} and N_{MC2} are adaptively chosen for each set to satisfy the equilibration condition.

III. PHYSICAL QUANTITIES

In SGs, the conventional order parameter is an overlap between two independent systems with a common Hamiltonian. In the case of the XY model, each spin has two components and the spin overlap becomes a tensor with indices α and β ($\alpha, \beta = x, y$). We define the wavevector \mathbf{k} -dependent spin overlap as

$$q_{\alpha\beta}(\mathbf{k}) = \frac{1}{N} \sum_{i=1}^N S_{i\alpha}^{(1)} S_{i\beta}^{(2)} e^{i\mathbf{k} \cdot \mathbf{r}_i}, \quad (4)$$

where the superscripts (1) and (2) denote two independent systems with the same Hamiltonian. For simplicity

of notation, we write

$$q_s(\mathbf{k}) = \sqrt{\sum_{\alpha,\beta} |q_{\alpha\beta}(\mathbf{k})|^2}. \quad (5)$$

Similarly, we define the chirality and introduce the associated overlap. The chirality at a plaquette p which is perpendicular to the $\mu(=x, y, z)$ axis is defined by

$$\kappa_{p\perp\mu} = \frac{1}{2\sqrt{2}} \sum_{\langle i,j \rangle \in p} \text{sgn}(J_{ij}) \sin(\theta_i - \theta_j), \quad (6)$$

where the directed sum $\sum_{\langle i,j \rangle \in p}$ is taken over four bonds surrounding the plaquette p in a clockwise direction^{47,67,68}. The chiral overlap is then given by

$$q_\kappa^\mu(\mathbf{k}) = \frac{1}{N} \sum_{p=1}^N \kappa_{p\perp\mu}^{(1)} \kappa_{p\perp\mu}^{(2)} e^{i\mathbf{k}\cdot\mathbf{r}_p}. \quad (7)$$

The SG order parameter q_{SG} and the SG susceptibility χ_{SG} are then defined by

$$q_{SG}^{(2)} = [\langle q_s(0)^2 \rangle], \quad \chi_{SG} = N q_{SG}^{(2)}, \quad (8)$$

where the square brackets $[\dots]$ denote the configurational average, *i.e.*, the average over the bond disorder.

The SG Binder parameter g_{SG} and the SG correlation length ξ_{SG} are defined by

$$g_{SG} = 3 - 2 \frac{[\langle q_s(0)^4 \rangle]}{[\langle q_s(0)^2 \rangle]^2}, \quad (9)$$

$$\xi_{SG} = \frac{1}{2 \sin(k_{\min}/2)} \sqrt{\frac{[\langle q_s(0)^2 \rangle]}{[\langle q_s(\mathbf{k}_{\min})^2 \rangle]} - 1}, \quad (10)$$

where $\mathbf{k}_{\min} = (2\pi/L, 0, 0)$. Note that the OZ form of the correlation is implicit in this definition of the correlation length.

On the other hand, the CG order parameter and the CG susceptibility are given by

$$q_{CG}^{(2)} = [\langle q_\kappa^\mu(0)^2 \rangle], \quad \chi_{CG} = N q_{CG}^{(2)}. \quad (11)$$

The direction μ -dependence of the right-hand side should vanish after the sample average $[\dots]$, and we take the average over $\mu = x, y, z$ in the actual calculation. The CG Binder parameter and the CG correlation length are given by

$$g_{CG} = \frac{1}{2} \left(3 - \frac{[\langle q_\kappa^\mu(0)^4 \rangle]}{[\langle q_\kappa^\mu(0)^2 \rangle]^2} \right), \quad (12)$$

$$\xi_{CG}^\mu = \frac{1}{2 \sin(k_{\min}/2)} \sqrt{\frac{[\langle q_\kappa^\mu(0)^2 \rangle]}{[\langle q_\kappa^\mu(\mathbf{k}_{\min})^2 \rangle]} - 1}. \quad (13)$$

Although the μ -dependence again vanishes for g_{CG} , it remains for ξ_{CG}^μ due to the nontrivial wavevector dependence of $q_\kappa^\mu(\mathbf{k}_{\min})$, *i.e.*, the dependence on the direction μ with respect to $\mathbf{k}(\parallel \hat{x})$ taken here parallel with \hat{x} . We

denote ξ_{CG}^x as ξ_{CG}^\parallel and $\xi_{CG}^{y,z}$ as ξ_{CG}^\perp , and will show both the data below.

We also define a parameter quantifying the non-self-averageness of the order parameter, the A parameter⁸³. It is defined either for the spin or for the chirality by

$$A_{SG} = \frac{[\langle q_s(0)^2 \rangle]^2 - [\langle q_s(0)^2 \rangle]^2}{[\langle q_s(0)^2 \rangle]^2}, \quad (14)$$

$$A_{CG} = \frac{[\langle q_\kappa^\mu(0)^2 \rangle]^2 - [\langle q_\kappa^\mu(0)^2 \rangle]^2}{[\langle q_\kappa^\mu(0)^2 \rangle]^2}. \quad (15)$$

The μ -dependence vanishes for A_{CG} . The A parameter becomes nonzero if the SG or the CG susceptibility is non-self-averaging. Note that, even when q_{SG} vanishes in the thermodynamic limit, A_{SG} can become finite if χ_{SG} is non-self-averaging. In the current problem, such a situation can emerge in the temperature range $T_{SG} < T < T_{CG}$ in the possible occurrence of the spin-chirality decoupling.

We also introduce the so-called Guerra parameter or the G parameter defined by

$$G_{SG} = \frac{[\langle q_s(0)^2 \rangle]^2 - [\langle q_s(0)^2 \rangle]^2}{[\langle q_s(0)^4 \rangle] - [\langle q_s(0)^2 \rangle]^2}, \quad (16)$$

$$G_{CG} = \frac{[\langle q_\kappa^\mu(0)^2 \rangle]^2 - [\langle q_\kappa^\mu(0)^2 \rangle]^2}{[\langle q_\kappa^\mu(0)^4 \rangle] - [\langle q_\kappa^\mu(0)^2 \rangle]^2}. \quad (17)$$

The G parameter looks like the A parameter, but there is a difference in that the G parameter can be finite even when the ordered state does not accompany the RSB^{84–86}.

The distributions of the spin and the chiral overlaps might provide a signal of the RSB. In this paper, we examine the following two overlap distributions

$$P_s(q) = \left[\delta \left(q - \sum_{\alpha} q_{\alpha\alpha}(0) \right) \right], \quad (18)$$

$$P_\kappa(q) = [\delta(q - q_\kappa^\mu(0))]. \quad (19)$$

For the chiral overlap distribution $P_\kappa(q)$, the RSB effect is simple: if there is no RSB in the ordered state, the distribution has only two δ -peaks in the thermodynamic limit which are related each other by the Z_2 spin-reflection symmetry of the whole spins. On the other hand, the spin overlap distribution $P_s(q)$ takes a non-trivial form even in the SG ordered state without the RSB, a superposition of two δ -peaks located at $q = \pm q_{EA}$ and a broad distribution spanning between these diverging peaks, due to the projection of the tensor $q_{\alpha\beta}$ onto the diagonal component. This makes obtaining a clear indication of the RSB from the P_s data rather difficult. For further details, see Ref. 47.

IV. MONTE CARLO RESULTS

In this section, we present the result of our MC simulations. We show first the temperature dependence of the specific heat in Fig. 1. No appreciable anomaly is seen in the specific heat, though the SG and the CG transition points actually exist in this temperature range, as will be shown below. The CG and the SG critical temperatures T_{CG} and T_{SG} are denoted by arrows in the figure.

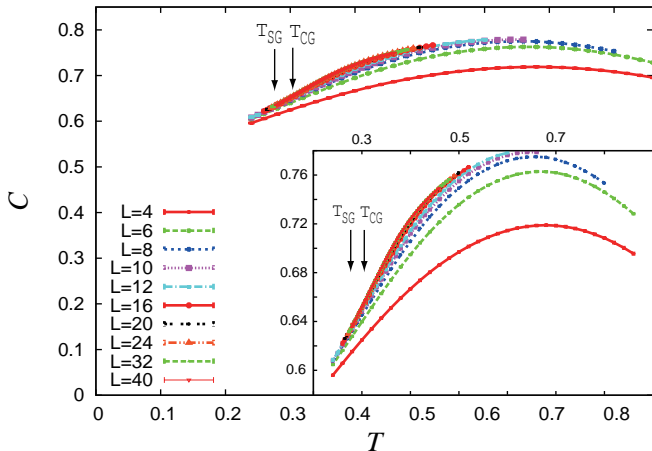


FIG. 1. (Color online) The temperature and size dependence of the specific heat per spin. Magnified view is given in the inset. The arrows at higher and lower temperatures indicate the CG and the SG transition points, respectively.

To investigate the SG and the CG orderings, we show the temperature dependence of the SG and the CG susceptibilities, χ_{SG} and χ_{CG} , in Fig. 2. In contrast to the SG susceptibility χ_{SG} , which tends to increase as the system size L is increased in the whole temperature region, the CG susceptibility χ_{CG} shows such a behavior only in the temperature region $T \lesssim 0.4$, whereas, in the region $T \gtrsim 0.4$, it exhibits an opposite size dependence. This implies that the CG critical region is relatively narrow, which is common with the observation of the 3D Heisenberg SG⁵⁵.

In Fig. 3, we plot the size dependence of the SG and the CG order parameters $q_{SG}^{(2)}$ and $q_{CG}^{(2)}$ for several temperatures on a double-logarithmic plot. The data of the CG order parameter $q_{CG}^{(2)}$ exhibit a straight-line behavior around a temperature $T \sim 0.306$. It exhibits a clear upward trend at lower temperatures implying the appearance of the CG long-range order, whereas at higher temperatures it exhibits a downward trend, eventually approaching another straight line with a slope $-d = -3$ generally expected in the disordered phase for large enough systems. Then, we get our first estimate of the CG transition temperature, $T_{CG} \sim 0.31$. By contrast, $q_{SG}^{(2)}$ exhibits such an upward trend only at the lowest temperature studied, $T = 0.266$, with a straight-line behavior observed around $T \sim 0.276$. Then, we get an estimate of the SG transition temperature, $T_{SG} \sim 0.28$.

Hence, our data of the size dependence of the glass order parameters $q^{(2)}$ suggest that the spin-chirality decoupling really occurs in the present model.

In Fig. 4, we show the SG and the CG Binder parameters. Consistently with the earlier reports^{46,47}, the CG Binder parameter exhibits a non-divergent dip and a crossing among different sizes on the negative side of g_{CG} . Such a behavior is expected in a system exhibiting a continuous one-step RSB (1RSB). The crossing and the dip temperatures are expected to converge to T_{CG} in the thermodynamic limit, which might provide a way to precisely estimate T_{CG} . By contrast, the SG Binder parameter g_{SG} shows no crossing nor dip, decreasing monotonically as the system size is increased. So far, information concerning the transition points has been hard to obtain from g_{SG} . We re-examine the relation of g_{SG} to the CG and the SG transition points below in appendix, and point out a possibility to extract information about T_{SG} and T_{CG} from the g_{SG} data.

In Fig. 5, we show the temperature dependence of the SG correlation-length ratio ξ_{SG}/L . Those of the parallel CG correlation-length ratio ξ_{CG}^{\parallel}/L and of the perpendicular CG correlation-length ratio ξ_{CG}^{\perp}/L are given in Fig. 6. Both the SG and the CG correlation-length ratios show crossing among different sizes, and the crossing temperatures are expected to converge to the corresponding critical temperatures in the $L \rightarrow \infty$ limit.

To estimate T_{SG} , we plot in the left panel of Fig. 7 the crossing temperatures of the SG correlation-length ratio ξ_{SG}/L for pairs of sizes L and sL with $s = 2, 3/2, 4/3, 5/3$ and $5/4$ versus $1/L_{ave}$ where $L_{ave} = (1+s)L/2$. Note that the crossing temperature for the pair $(L, sL) = (32, 40)$ with $s = 5/4$ is estimated by extrapolating the data to lower temperatures, since the raw data of ξ_{SG}/L do not show a crossing in the investigated temperature range.

We then try an infinite-size extrapolation of the crossing temperature $T_{cross}(L; s)$ based on the scaling from,

$$T_{cross}(L; s) = T_c + c_s L^{-\theta}, \quad \theta = \omega + \frac{1}{\nu}. \quad (20)$$

In the fit of $T_{cross}(L; s)$ of the SG correlation-length ratios ξ_{SG}/L , we perform a combined fit of different s -sequences with a common $T_c = T_{SG}$ and a common $\theta = \theta_{SG}$. We use in the fit the data for $s = 2, 3/2, 4/3$ and $5/3$ only, not the one for $s = 5/4$ because of a possible inaccuracy due to the extrapolation employed for the data of $(L, sL) = (32, 40)$ mentioned above. The resulting fitting curves are shown in the left panel of Fig. 7. The optimal fit is obtained for $T_{SG} = 0.274$ and $\theta_{SG} = 1.1$. To estimate the associated error bar, we plot in the inset of Fig. 8 the total χ -square value of the fit versus the assumed T_{SG} value. The horizontal line in the figure represents the total χ -square value greater than the optimal value observed at $T_{SG} = 0.274$ by unity, which gives our error criterion. The asymmetry of the curve results in the different values for the upper and the lower error values. We then quote $T_{SG} = 0.274^{+0.012}_{-0.032}$ and $\theta_{SG} = 1.10^{+0.60}_{-0.55}$.

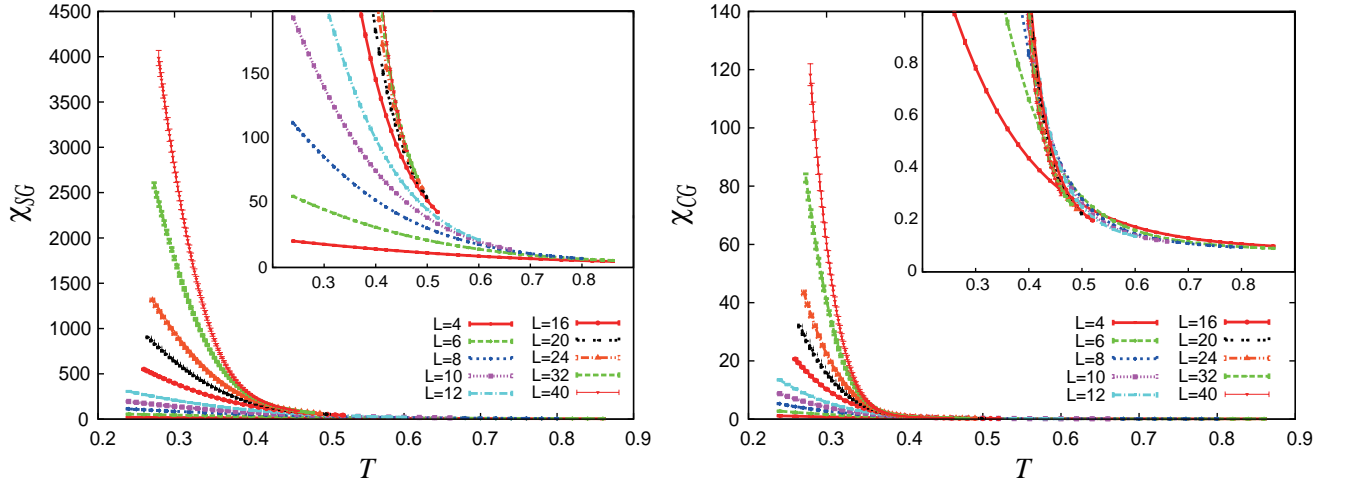


FIG. 2. (Color online) The temperature and size dependence of the SG susceptibility χ_{SG} (left), and of the CG susceptibility χ_{CG} (right). The insets are magnified views. As can be seen from the insets, the magnitude of χ_{SG} increases as the system size is increased in the whole temperature region, while that of χ_{CG} exhibits an opposite size dependence in the temperature region $T \gtrsim 0.4$.

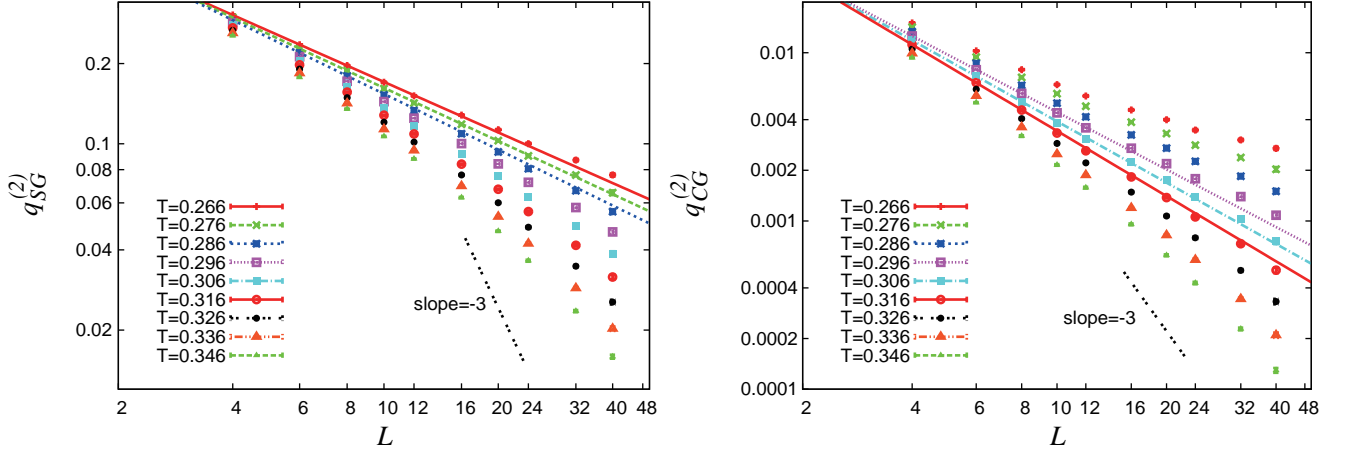


FIG. 3. (Color online) The size dependence of the SG order parameter (left), and of the CG order parameter (right) on a log-log plot for several temperatures. Straight lines are drawn by fitting the three data points of smaller sizes of $L = 4, 6, 8$. The $L = 24, 32$ data for $T = 0.266$ as well as the $L = 40$ data for $T = 0.266$ and 0.276 are obtained by extrapolating the higher temperature data to lower temperatures. A line with a slope $-d = -3$, which is the expected large- L asymptotic behavior in the disordered phase, is also drawn.

Next we turn to the estimate of T_{CG} based on the CG correlation-length ratio ξ_{CG}/L . Although the parallel one ξ_{CG}^{\parallel}/L and the perpendicular one ξ_{CG}^{\perp}/L give two different sequences of the crossing temperatures, they tend to accord for $L_{ave} \gtrsim 8$ as can be seen in the right panel of Fig. 7 (see also the inset). Hence, to extract T_{CG} , we use the data of ξ_{CG}^{\perp} for $L_{ave} \geq 8$ only. Unfortunately, and somewhat unexpectedly, the result of the fit of ξ_{CG}/L turns out to be rather pathological. The estimated best T_{CG} -value becomes extremely small or even negative. The resultant fitting curves are shown in the right panel of Fig. 7 (the lower curves). We also examine the possible change in the fit with varying the lowest

size used in the fit, but the pathology cannot be cured. The cause of this pathology is not entirely clear, but may be due to the peculiar behavior of the chiral-glass correlations. For example, χ_{CG} shows a non-monotonic size dependence absent in the corresponding χ_{SG} . To further clarify this point, we discuss the properties of the spatial CG correlation function below in this section. Anyway, in the present analysis, we abandon the crossing temperatures of ξ_{CG}/L in estimating T_{CG} .

In estimating T_{CG} , the crossing and the dip temperatures of g_{CG} can also be used, and the data are plotted in the right panel of Fig. 7. The crossing temperature of g_{CG} obeys the scaling form Eq. (20) with $T_c = T_{CG}$ and $\theta = \theta_{CG} = \omega_{CG} + 1/\nu_{CG}$, whereas the dip temperature

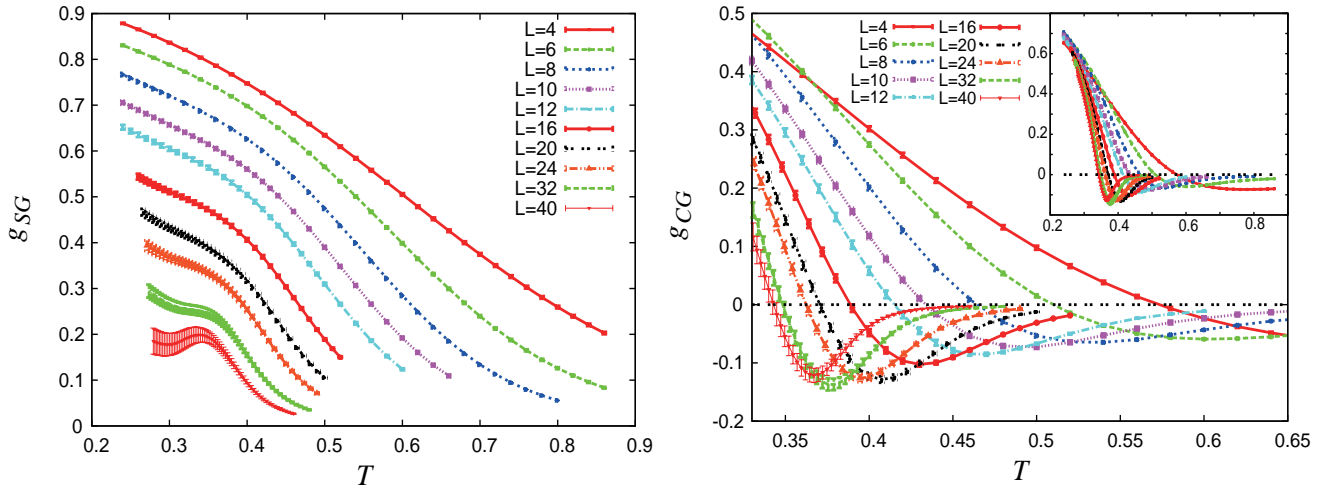


FIG. 4. (Color online) The temperature and size dependence of the SG Binder parameter g_{SG} (left), and of the CG Binder parameter g_{CG} (right). The inset of the right panel exhibits g_{CG} in the wider temperature range.

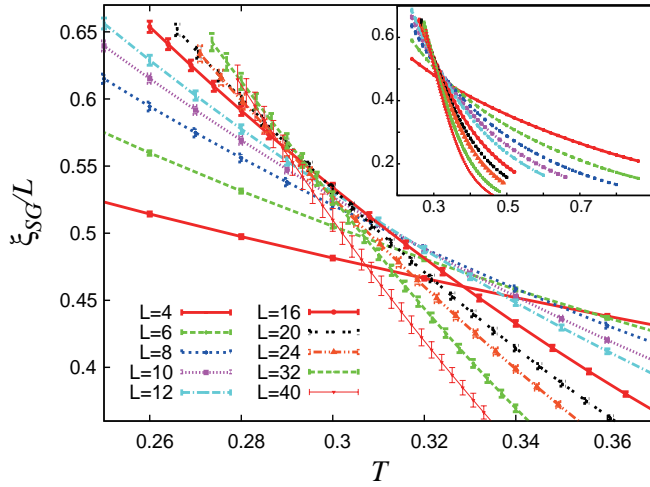


FIG. 5. (Color online) The temperature and size dependence of the SG correlation-length ratio ξ_{SG}/L . The inset exhibits the wider temperature range.

is expected to follow the scaling form,

$$T_{dip}(L) = T_{CG} + c_1 L^{-\frac{1}{\nu_{CG}}} + c_2 L^{-\theta_{CG}}. \quad (21)$$

Conventionally, the sub-leading correction-to-scaling term, $c_2 L^{-\theta_{CG}}$, is dropped because it gives a relatively smaller contribution than that of the leading correction-to-scaling term $c_1 L^{-1/\nu_{CG}}$ for larger L . In the range of system sizes of our simulations, however, we need to take into account this correction term for describing the size dependence of T_{cross} , since the leading term $c_1 L^{-1/\nu_{CG}}$ should describe the increase of the dip temperature with the system size, which is not observed in our simulation. (Such an increase of $T_{dip}(L)$ was indeed observed in a recent simulation of the 4D Heisenberg SG⁸⁷.) The behavior originates from the fact that both the dip and

the crossing temperatures of g_{CG} should converge to a common value, T_{CG} , each with an exponent $1/\nu$ and $\theta(> 1/\nu)$, while the crossing temperature always lies above the dip one.

The combined fit of the crossing and the dip temperatures of g_{CG} based on Eqs. (20,21) with a common T_{CG} and a common θ_{CG} yields $T_{CG} = 0.308$ and $\theta_{CG} = 0.88$. In our estimate of error bars, we again calculate the total χ -square value against the assumed T_{CG} value, and the result is given in the main panel of Fig. 8. Our estimate of the CG transition temperature is then $T_{CG} = 0.308 \pm 0.005$ and that of the exponent is $\theta_{CG} = 0.88 \pm 0.03$. Note that the leading term in Eq. (21) tends to be masked by the correction term, *i.e.*, c_1 tends to be considerably smaller than c_2 , and the value of ν_{CG} is less precise.

The SG and the CG transition temperatures $T_{SG} = 0.274^{+0.012}_{-0.032}$ and $T_{CG} = 0.308 \pm 0.005$ estimated from the Binder parameter are well consistent with the values obtained from the order parameter given in Fig. 3. Our error analysis indicates that T_{SG} and T_{CG} are indeed separate. These observations certainly speak for the occurrence of the spin-chirality decoupling in the 3D XY SG. The difference in the CG and the SG transition temperatures is about 10 percent, which is comparable with the corresponding value of the 3D Heisenberg SG⁵⁵.

In Fig. 9, we plot the ratio of the CG and the SG correlation lengths, ξ_{CG}/ξ_{SG} . For smaller sizes of $6 \leq L \leq 12$, the ratio curves are almost size-independent⁶⁷ at lower temperatures, while for intermediate sizes of $16 \leq L \leq 24$ the curves start to splay out but the tendency is still small⁶⁸. For larger sizes of $L = 32$ and 40 , the tendency becomes stronger and the ratio exceeds unity at low temperatures, which supports the spin-chirality decoupling ansatz. The intersection point of the ratio curves between different sizes lie around $T \sim 0.31$, which is consistent with our estimate of $T_{CG} = 0.308 \pm 0.05$.

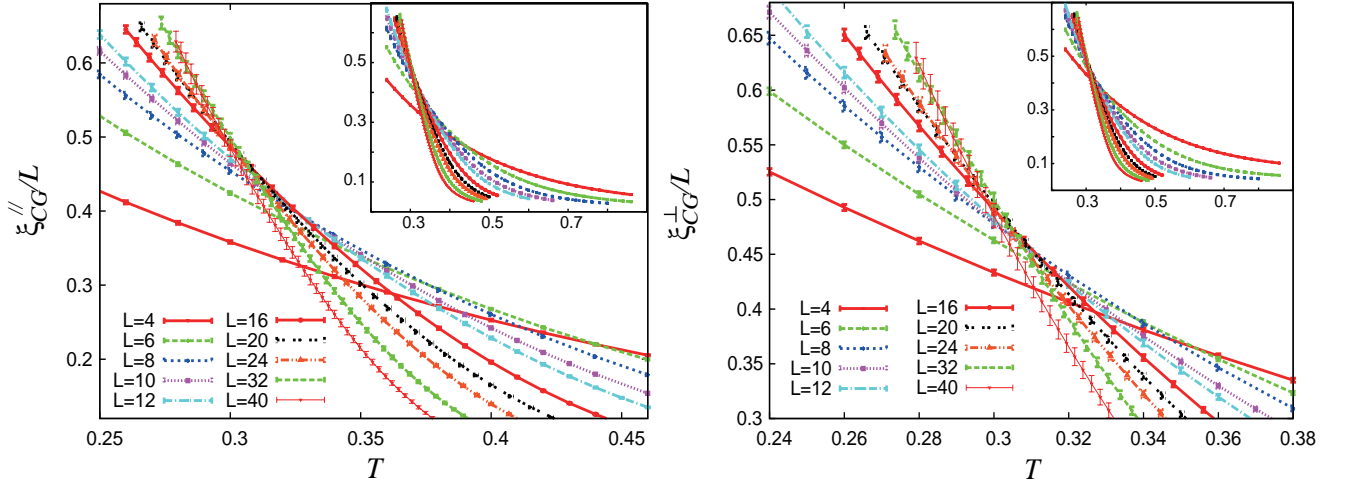


FIG. 6. (Color online) The temperature and size dependence of the CG parallel correlation-length ratio ξ_{CG}^{\parallel}/L (left), and of the CG perpendicular correlation-length ratio ξ_{CG}^{\perp}/L (right). The insets exhibit the wider temperature range.

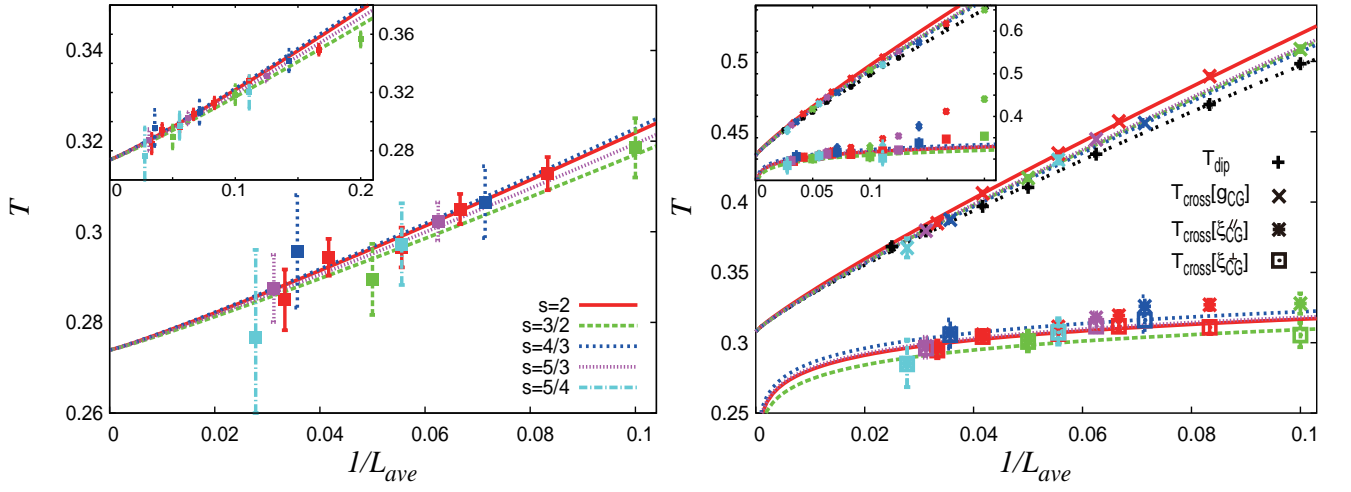


FIG. 7. (Color online) The crossing temperatures T_{cross} of the SG correlation-length ratio ξ_{SG}/L of the two sizes L and sL are plotted versus the inverse mean lattice size $1/L_{ave}$ where $L_{ave} = (1+s)L/2$ (left). The crossing temperatures of the CG perpendicular correlation-length ratio ξ_{CG}^{\perp}/L and of the CG Binder parameter g_{CG} as well as the dip temperature T_{dip} of g_{CG} are plotted versus $1/L_{ave}$ (or $1/L$) (right). The insets exhibit the wider size range.

Now, to get further insight into the cause of the pathological behavior we encountered in ξ_{CG} , we discuss the CG and the SG spatial correlations, particularly how they obey to or deviate from the standard OZ form. The SG and the CG spatial correlation functions, $C_s(x)$ and $C_{\kappa}(x)$, are defined by

$$C_s(|\mathbf{r}_i - \mathbf{r}_j|) = \sum_{\alpha, \beta} \left[\left\langle S_{i\alpha}^{(1)} S_{i\beta}^{(2)} S_{j\alpha}^{(1)} S_{j\beta}^{(2)} \right\rangle \right], \quad (22)$$

$$C_{\kappa}(|\mathbf{r}_p - \mathbf{r}_q|) = \left[\left\langle \kappa_{p\perp\mu}^{(1)} \kappa_{p\perp\mu}^{(2)} \kappa_{q\perp\mu}^{(1)} \kappa_{q\perp\mu}^{(2)} \right\rangle \right]. \quad (23)$$

The computed C_s and C_{κ} for $L = 16$ lattices under periodic BC are shown in Fig. 10 for several temperatures. Note that the CG correlation function C_{κ} is normalized by its local amplitude so as to give unity at $x = 0$. The

data are plotted on a semi-logarithmic scale so that the standard OZ form gives a straight line. The leveling-off of the data observed at larger x is a finite-size effect due to the imposed periodic BC. One can see from the figure that $C_{\kappa}(x)$ drops rapidly from unity in the small- x region of a few lattice spacings, strongly deviating from the OZ form, and becomes by an order of magnitude smaller than $C_s(x)$ for larger x even below T_{CG} . Such a sharp drop of the spatial correlations at short length is much less pronounced in $C_s(x)$. This feature of the CG correlations might cause some problems in defining the finite-size correlation length ξ_{CG} based on the OZ form, at least for small lattices treated in our present simulation. This is because the OZ-based definition of ξ_{CG} , Eq. (13), contains the $k = 0$ part $[\langle q_{\mu}^{\mu}(0)^2 \rangle]$, which is essen-

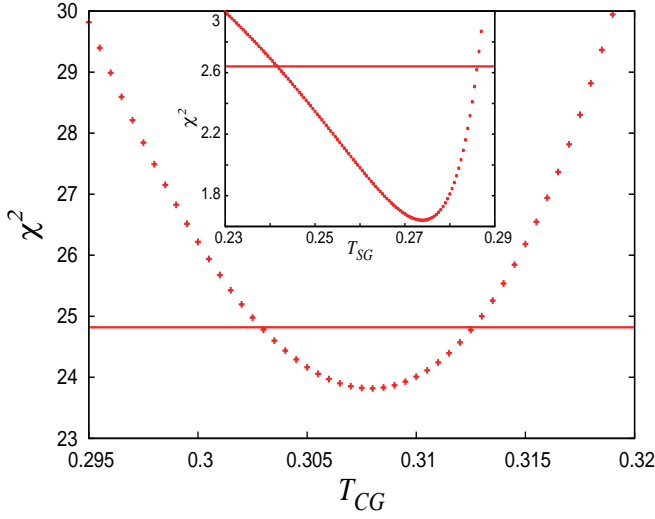


FIG. 8. (Color online) The total χ -square values associated with the combined fit of the crossing and dip temperatures of the CG binder parameters g_{CG} are plotted versus the CG transition temperature T_{CG} assumed in the fit. The horizontal line represents the total χ -square value greater than the optimal value by unity, usually used as an error criterion. The corresponding plot of the total χ -square values obtained in estimating T_{SG} is also presented in the inset.

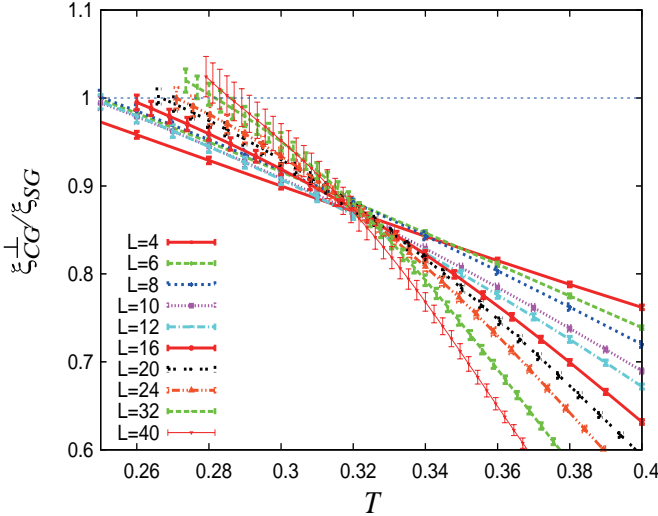


FIG. 9. (Color online) The temperature and size dependence of the ratio of the CG and the SG correlation lengths $\xi_{CG}^\perp / \xi_{SG}$.

tially an equal-weight sum of CG correlation functions, $\int dx C_\kappa(x)$. Since $C_\kappa(x)$ in the large- x region, which should govern the true CG correlation length, is much smaller in magnitude than that in the small- x region, the latter contribution not playing an essential role in the CG correlation length might make a major contribution to $[\langle q_\kappa^\mu(0)^2 \rangle]$, and mask or obscure the asymptotic behavior of the CG correlation length. Thus, the standard OZ form may be inappropriate to treat the CG correla-

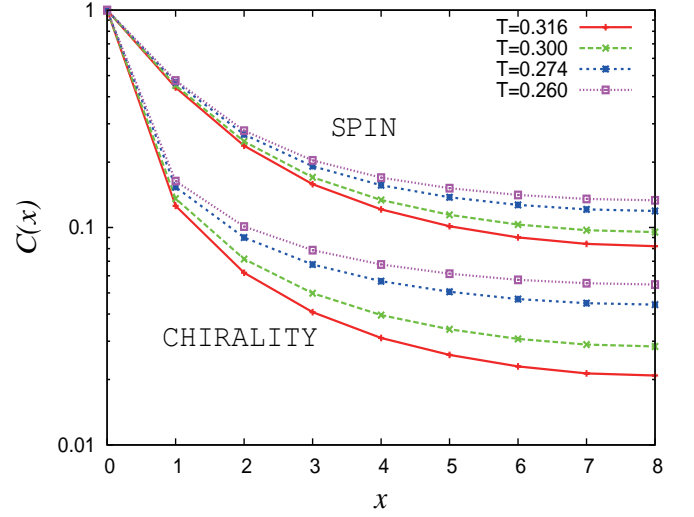


FIG. 10. (Color online) The spatial correlation functions of the SG and of the CG overlaps for several temperatures around $T_{CG} \approx 0.308$ and $T_{SG} \approx 0.274$. The upper curves are for the spin overlap and the lower ones are for the chiral one. The lattice is $L = 16$ under periodic BC. Total number of samples is $N_s = 500$. Error bars are omitted. Note the logarithmic scale of the ordinate.

tion length, particularly when the system size is small. A similar observation was also made in the 4D Ising SG in a magnetic field⁸⁸. To overcome this difficulty, another dimensionless quantity was already proposed⁸⁸, but the examination of the quantity is beyond the scope of the present paper.

Next, we turn to the quantities which probe the phase-space structure of the ordered state, including the overlap distribution and the non-self-averageness parameter. In Fig. 11, we show the distributions of the spin and the chiral overlaps at a temperature $T = 0.2792$, which lies below T_{CG} and very close to (slightly above) T_{SG} . For $L \geq 10$, the chiral-overlap distribution P_κ shown in the right panel of Fig. 11 exhibits a central peak in addition to the side peaks corresponding to the CG EA order parameter $\pm q_{CG}^{EA}$. As the system size L increases, all the peaks grow in their height and become narrower in their width. This implies that all these peaks would remain in the thermodynamic limit. These features are nothing but the character of the 1RSB, and are consistent with the occurrence of a negative dip in g_{CG} . Similar behaviors were observed in several types of Heisenberg and XY SGs before^{47,51,55}, but the side peaks observed in our present simulation for the XY SG seem sharper than those observed in the Heisenberg SG.

By contrast, the spin-overlap distribution shown in the left panel of Fig. 11 exhibits a shoulder-like structure only for small sizes, which tends to be suppressed as the system size increases. As the growing side peaks located at $\pm q_{SG}^{EA}$ are expected in the SG ordered phase⁴⁷, this observation is consistent with the absence of the SG order at this temperature $T = 0.2792$, which is indeed compatible

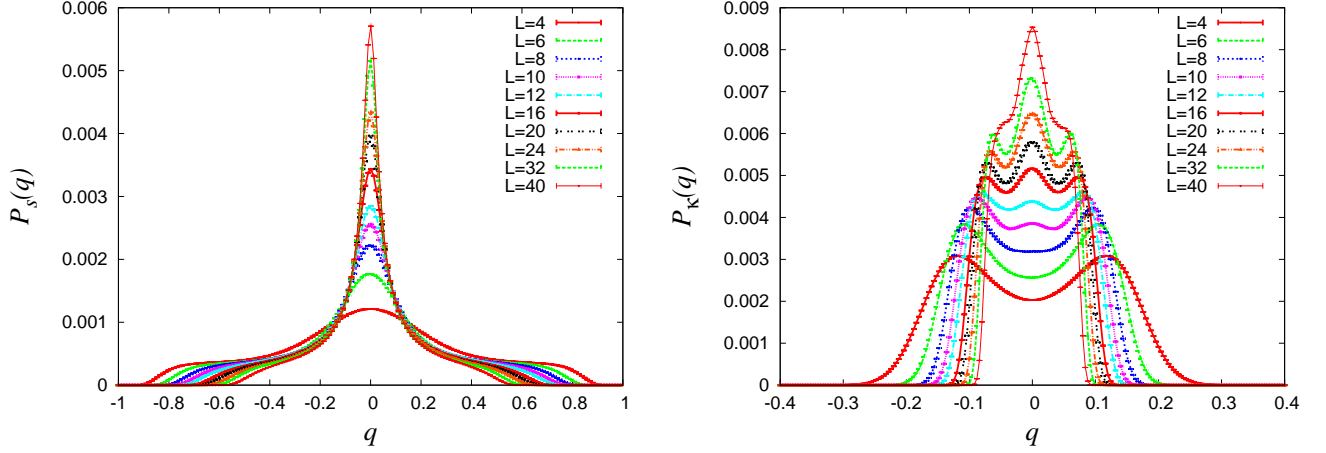


FIG. 11. (Color online) The spin diagonal overlap distribution (left), and the chiral overlap distribution (right), at a temperature $T = 0.2792$ which lies in the CG ordered state, *i.e.* below $T_{CG} \simeq 0.308$ but slightly above $T_{SG} \simeq 0.274$. A typical 1RSB behavior is observed in the chiral overlap distribution.

with our estimate above, $T_{SG} = 0.274^{+0.012}_{-0.032}$.

In Fig. 12, the SG and CG non-self-averageness A parameters are plotted against the temperature for various system sizes. As can be seen from the figure, A_{CG} of different sizes intersect around $T \sim T_{CG}$. A prominent peak is observed on the lower temperature side, which grows as the system size increases. This suggests that the self-averageness of the system is broken below T_{CG} and that the CG ordered phase is non-self-averaging. This is quite consistent with the 1RSB nature of the CG phase, as already signaled by the central peak in P_κ and by the negative dip of g_{CG} . The parameter A_{SG} exhibits a behavior similar to A_{CG} , accompanied with an intersection and a peak around $T = T_{CG}$, even though the SG order is still absent at $T = T_{CG}$. As already noted in Sec. III, this is not surprising since a finite A_{SG} just means the non-self-averageness of χ_{SG} which originates from the CG transition involving the phase-space narrowing associated with the 1RSB. Hence, the intersection and the peak of A_{SG} around T_{CG} is completely compatible with the spin-chirality decoupling.

We also show the SG and CG G parameters in Fig. 13. The observed behaviors are quite similar to the corresponding ones of the A parameters. Note that, although the intersections of the A and G parameters can also be used in estimating the transition temperature in principle, the data tend to be noisy and not suited to precisely locate T_{CG} . The same suggestion was made for the 3D Ising SG^{7,89} and for the 3D Heisenberg SG⁵⁵.

V. CRITICAL PROPERTIES

In this section, we study the critical properties of the SG and the CG transitions based on the finite-size scaling analysis. To estimate the CG critical exponents, we use both the CG Binder parameter and the CG susceptibility. On the other hand, we use the SG susceptibility only to

estimate the SG critical exponents, the reason of which will be explained below.

Let us start from the CG criticality. The standard finite-size scaling forms of the CG Binder parameter g_{CG} and of the CG susceptibility χ_{CG} are given by

$$g_{CG} = \tilde{X} \left((T - T_{CG}) L^{1/\nu_{CG}} \right), \quad (24)$$

$$\chi_{CG} = L^{2-\eta_{CG}} \tilde{Y} \left((T - T_{CG}) L^{1/\nu_{CG}} \right), \quad (25)$$

where \tilde{X} and \tilde{Y} are appropriate scaling functions.

For the CG susceptibility χ_{CG} , a good data collapse can be obtained by the two-parameter fits based on Eq. (25), with $\nu_{CG} = 1.3$ and $\eta_{CG} = 0.23$. Note that these values are obtained via the Bayesian scaling analysis (BSA), which enables us to estimate the critical exponents in an unbiased way⁹⁰, after fixing the transition temperature to the value obtained in the previous section $T_{CG} = 0.308$. By changing the assumed value of T_{CG} in the range of the associated error bar, we obtain the error bars of ν_{CG} and η_{CG} as $\nu_{CG} = 1.3 \pm 0.1$ and $\eta_{CG} = 0.23 \pm 0.22$.

The crossing temperature of the CG Binder parameter g_{CG} still exhibits appreciable size dependence, indicating the necessity to invoke the correlation-to-scaling term in the finite-size scaling. With the correction term, the scaling form of g_{CG} is modified as

$$g_{CG} = \tilde{X} \left((T - T_{CG}) L^{1/\nu_{CG}} \right) (1 + a L^{-\omega_{CG}}), \quad (26)$$

where a is a numerical constant. The BSA analysis based on this form yields $\nu_{CG} = 1.4 \pm 0.1$ and $\omega_{CG} = 0.46 \pm 0.12$, which is consistent with $\nu_{CG} = 1.3 \pm 0.1$ estimated above from the CG order parameter. These values lead to $(1/\nu_{CG}) + \omega_{CG} \approx 1.2$, which is slightly larger than, but not incompatible with $\theta_{CG} = 0.88 \pm 0.03$ obtained in the previous section. The resultant finite-size scaling plot is given in the left panel of Fig. 14.

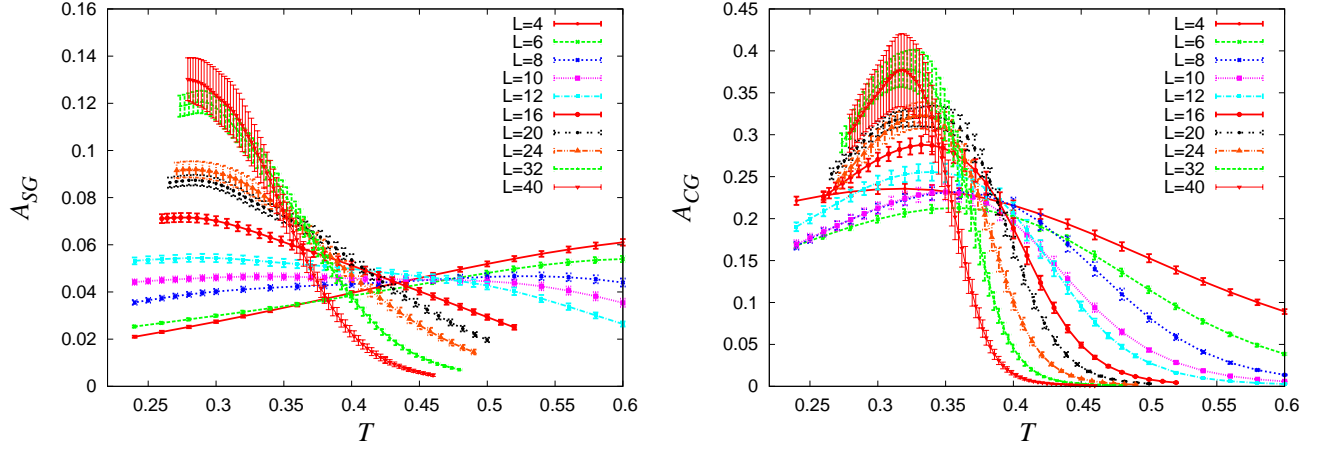


FIG. 12. (Color online) The temperature and size dependence of the non-self-averageness A parameters of the SG (left), and of the CG (right).

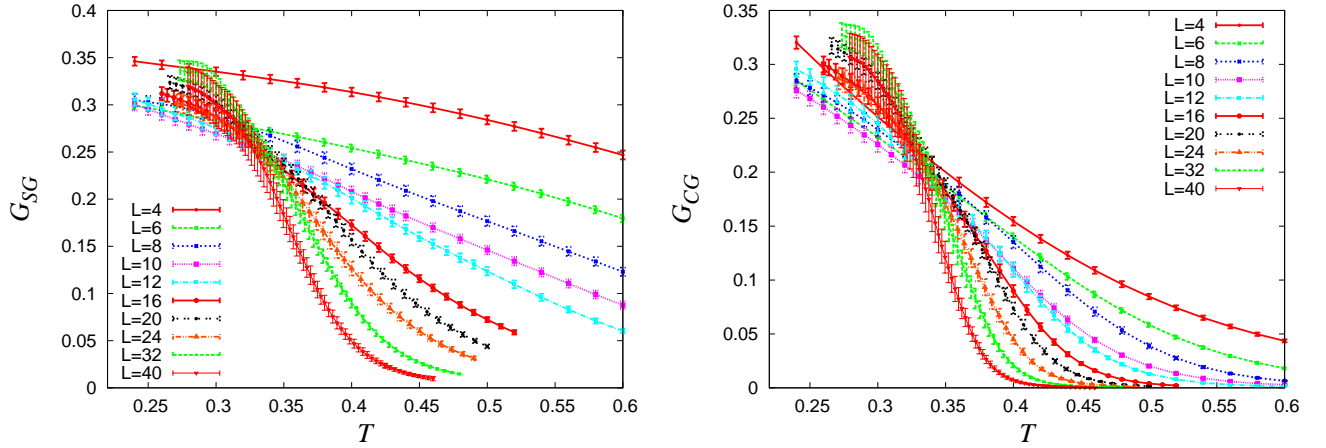


FIG. 13. (Color online) The temperature and size dependence of the G parameters of the SG (left), and of the CG (right).

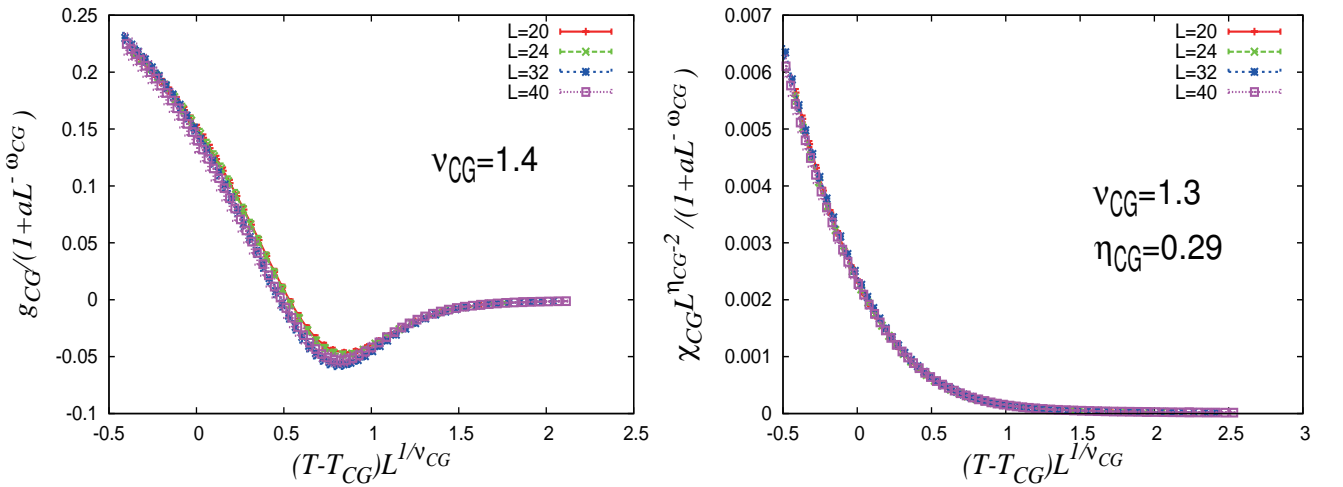


FIG. 14. (Color online) Finite-size-scaling plots of the CG Binder parameter (left), and of the CG susceptibility (right), where the correction-to-scaling effect is taken into account. The CG transition temperature is fixed to $T_{CG} = 0.308$. Best fit is obtained for $\nu_{CG} = 1.4$ and $\omega_{CG} = 0.46$ for g_{CG} (left), and $\nu_{CG} = 1.3$, $\eta_{CG} = 0.29$ and $\omega_{CG} = 0.46$ for χ_{CG} (right).

We examine the scaling form with the correction term also for the CG susceptibility,

$$\chi_{CG} = L^{2-\eta_{CG}} \tilde{Y} \left((T - T_{CG}) L^{1/\nu_{CG}} \right) (1 + aL^{-\omega_{CG}}). \quad (27)$$

Based on this form, we get $\nu_{CG} = 1.3 \pm 0.1$ and $\eta_{CG} = 0.29 \pm 0.12$. These values are consistent with the values obtained above without invoking the correction term. The resultant finite-size scaling plot is given in the right panel of Fig. 14.

Next, we move to the SG criticality. As in the CG case, the standard scaling form of the SG susceptibility is given by

$$\chi_{SG} = L^{2-\eta_{SG}} \tilde{Y} \left((T - T_{SG}) L^{1/\nu_{SG}} \right). \quad (28)$$

A good data collapse of the SG susceptibility is obtained based on this form, with the resultant exponents $\nu_{SG} = 1.23^{+0.17}_{-0.06}$ and $\eta_{SG} = -0.42^{+0.12}_{-0.27}$. The associated scaling plot is given in Fig. 15. Although we can obtain

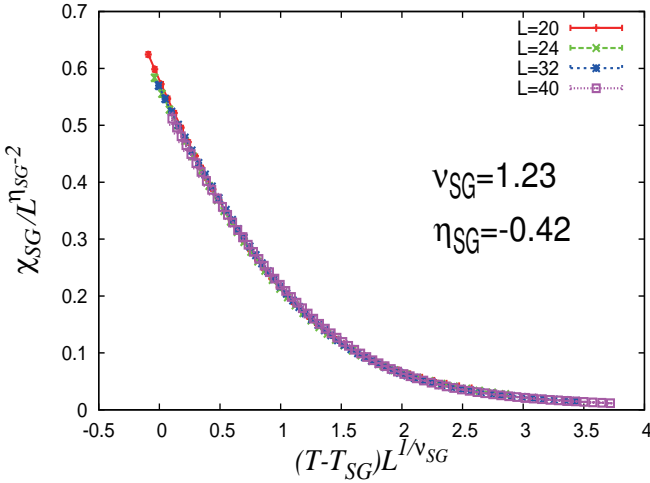


FIG. 15. (Color online) Finite-size-scaling plot of the SG susceptibility without the correction-to-scaling term. The SG transition temperature is fixed to $T_{CG} = 0.274$. Best fit is obtained for $\nu_{SG} = 1.23$ and $\eta_{SG} = -0.42$.

an estimate of ω_{SG} by combining the value of ν_{SG} and that of $\theta_{SG} = 1.10^{+0.60}_{-0.55}$ estimated in the previous section, the result is inaccurate and poor, unfortunately. Other quantities such as the SG correlation length or the SG Binder parameter do not resolve this problem. Indeed, we also have examined the finite-size scaling of ξ_{SG}/L with the correction term, but ended up with an unphysical result of negative ω_{SG} . This inadequacy may partly be due to the fact that the estimated T_{SG} is located out of the range of the simulated temperature range.

Summarizing the above results, we finally quote as our best estimates of the CG exponents as

$$\nu_{CG} = 1.4 \pm 0.1, \quad \eta_{CG} = 0.29 \pm 0.12, \quad (29)$$

and the SG exponents as

$$\nu_{SG} = 1.23^{+0.17}_{-0.06}, \quad \eta_{SG} = -0.42^{+0.12}_{-0.27}. \quad (30)$$

The estimated CG critical exponents are compatible with the ones reported in earlier literature on the 3D XY SG^{46,47,64,66,68}. They are also quite close to the values of the 3D Heisenberg SG⁵⁵, whereas they are largely different from the values of the 3D Ising SG, $\nu = 2.5 \sim 2.7$ and $\eta = -0.38 \sim -0.40$ ^{10,11} in spite of a common Z_2 symmetry between the Ising spin and the chirality of the present model. The types of the RSB in the Ising SG and that in the XY or the Heisenberg SG may explain this difference, *i.e.*, the full RSB in the former versus the 1RSB in the latter. The phase-space structure of the XY and the Heisenberg SGs is essentially different from that of the Ising SG. Such speculation also leads to another question: what causes the difference in the RSB types among the Ising, the XY and the Heisenberg SGs? A possible explanation might be that the chirality-chirality interaction has a long-range nature different from the Ising one. Further study is needed to clarify these points.

VI. SUMMARY AND DISCUSSIONS

In this paper, we studied equilibrium ordering properties of the 3D isotropic XY SG by means of extensive MC simulations, up to the linear size $L = 40$. Examining various physical quantities including the glass order parameter, the Binder parameter, the correlation-length ratio and the overlap distribution function, we succeeded in giving reasonable numerical evidence that the SG and the CG transitions occur at two different temperatures. The estimated SG and CG critical temperatures are $T_{SG} = 0.274^{+0.012}_{-0.032}$ and $T_{CG} = 0.308 \pm 0.005$, respectively, and the difference in the two transition temperatures is about 10 percent, which is comparable with the difference observed in the 3D Heisenberg SG. Our conclusion of the occurrence of the spin-chirality decoupling in the model is in apparent contrast to that of the recent simulation by Pixley and Young⁶⁸. The main cause of the difference is that their analysis was mainly based on the correlation-length ratios. We have also confirmed that, as observed by Pixley and Young, the crossing points of the CG correlation-length ratio behave in not much different manner from the SG ones, which are seemingly consistent with a simultaneous SG and CG transition. Our present quantitative analysis, however, revealed that the extrapolated $T_{cross}(L)$ of the ξ_{CG}/L lead to an unphysical estimate of T_{CG} , *i.e.*, a negative one, whereas that of ξ_{SG}/L lead to a reasonable estimate of T_{SG} . It turned out that such a behavior of the CG correlation-length ratio is inconsistent with that of other quantities such as the CG order parameter and the CG Binder parameter. In particular, the size dependence of the glass order parameter speaks for the successive CG and SG transitions occurring at two different temperatures, as seen in Fig. 3. Our attitude is that the order parameter among various quantities is expected to give a stable and reliable result, particularly when it is corroborated by the Binder parameter as we observed here, since it is the most fun-

damental quantity in describing phase transition. We then argued that the observed ill behavior of the finite-size CG correlation length, which is defined based on the OZ form, may be due to the special character of the CG spatial correlations deviating significantly from the standard OZ form at short length and were susceptible to the pronounced finite-size effect.

The critical properties of the SG and the CG orderings were also examined by means of the finite-size-scaling analysis. By controlling the correction-to-scaling effect, we obtained the CG critical exponents as $\nu_{CG} = 1.4 \pm 0.1$ and $\eta_{CG} = 0.29 \pm 0.12$. These values are close to the corresponding Heisenberg SG values, but are quite different from the Ising SG values in spite of a common Z_2 symmetry. The SG critical exponents were also estimated to be $\nu_{SG} = 1.23^{+0.17}_{-0.06}$ and $\eta_{SG} = -0.42^{+0.12}_{-0.27}$, which were consistent with the earlier estimates.

The RSB nature of the ordered state was probed via the Binder parameter, the overlap distribution and the non-self-averageness parameter. All the quantities consistently point to the 1RSB in the model. Physical significance of the 1RSB feature was already discussed in the Heisenberg case from several perspectives³¹. It would also be interesting to examine the possible 1RSB properties in real materials related to the XY SG such as granular cuprate superconductors^{75–77,79,80}, together with the successive transitions and the associated critical properties.

As mentioned, the CG transition of the XY SG model and the Ising SG transition belong to different universality classes in spite of a common Z_2 symmetry between the chirality and the Ising spin variable. We speculate that this might originate from the difference in the type of the RSB in the two systems. The 3D Ising SG is believed to exhibit a full RSB^{6,83,84}, though some counter opinions also exist^{91–94}. By contrast, the 3D XY SG exhibits the 1RSB. These different types of RSB may be related to the difference in the critical properties of the Ising SG transitions and the CG transition of the XY SG. Furthermore, since such a 1RSB-like feature is also observed in the 3D Heisenberg SG⁵⁵, the above consideration suggests a possibility that the CG transitions of the XY and the Heisenberg SGs are actually the same, which are indeed consistent with our present numerical results.

ACKNOWLEDGMENTS

The authors are grateful to T. Okubo, G. Parisi and H. Yoshino for valuable discussion and comments. Especially, the discussion on the asymptotic form of the SG Binder parameter given in appendix is much owed to T. Okubo. G. Parisi also gave useful suggestions concerning the property of the CG spatial correlations argued in Sec. IV. This study is supported by Grant-in-Aid Scientific Research on Priority Areas ‘Novel State of Matter Induced by Frustration’ (19052006 and 19052008). We

thank ISSP, University of Tokyo, and YITP, Kyoto University for providing us with the CPU time.

Appendix A: The behavior of the spin-glass Binder parameter in the thermodynamic limit

In this appendix, we examine how the SG Binder parameter g_{SG} in the thermodynamic limit behaves across T_{CG} and T_{SG} in the occurrence of the spin-chirality decoupling.

In the CG phase realized at $T_{SG} < T < T_{CG}$, the average spin overlap $\langle q_{\alpha\beta} \rangle$ vanishes as in the high-temperature paramagnetic phase. Meanwhile, the Z_2 spin-reflection symmetry is spontaneously broken in the CG phase, which implies that the determinant of the spin-overlap tensor, $\det q_{\alpha\beta}$, takes a nonzero value. Denoting this symmetry-breaking bias coming from the CG order by $h(T)$, we may write the distribution of the spin-overlap tensor $q_{\alpha\beta}$ as

$$P(\{q_{\alpha\beta}\}) \propto e^{-\frac{N}{2\sigma^2}(\sum_{\alpha,\beta} q_{\alpha\beta}^2) - h(T) \det q_{\alpha\beta}}. \quad (A1)$$

Since the standard SG order is absent in the CG phase, the average of any simple moment such as $\langle q_{\alpha\beta}^2 \rangle$ vanishes in the thermodynamic limit, whereas that of the determinant remains to be nonzero. Using this distribution, the SG Binder parameter is calculated as

$$g_{SG} = -\frac{1}{4}h^2(T). \quad (A2)$$

Hence, we find that the SG Binder parameter takes a negative value in the region $T_{SG} < T < T_{CG}$ in spite of the absence of the long-range SG order. Although the detailed form of $h(T)$ is unclear, $h(T)^2$ would grow continuously and monotonically from zero when the temperature T is decreased across $T = T_{CG}$. It should be noted that, in the CG state of the 3D Heisenberg SG, g_{SG} still remains to be zero in sharp contrast to the XY SG case⁹⁵.

In the SG ordered state realized at $T < T_{SG}$, g_{SG} would take a different form. If there would be no RSB in the SG ordered state, g_{SG} in the thermodynamic limit would jump to unity below T_{SG} . If there occurs the 1RSB as in the present model, g_{SG} would take a nontrivial value not equal to unity below T_{SG} , eventually tending to unity in the $T \rightarrow 0$ limit. We show in Fig. 16 a schematic shape of g_{SG} in the thermodynamic limit expected in the present model.

Fig. 16 may enable us to extract further information about the transition temperatures from the data of the SG Binder parameter g_{SG} for larger sizes. We see several characteristic temperatures in g_{SG} for larger sizes, displayed in the left panel of Fig. 4. For example, we see in the data of $L = 40$ two extrema and an inflection point in between. In the thermodynamic limit, the extremum at a higher temperature would converge to T_{CG} , while the one at a lower temperature to T_{SG} . Unfortunately,

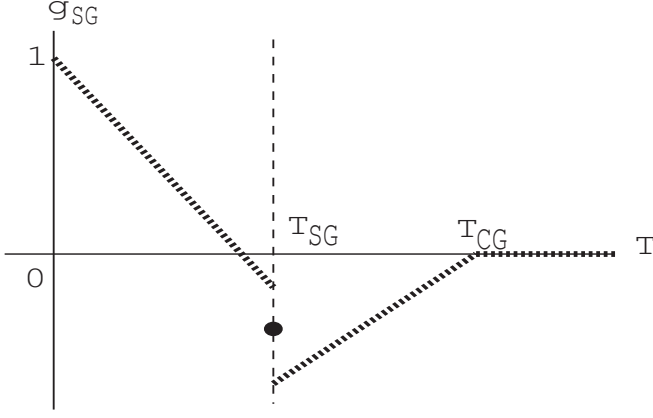


FIG. 16. A schematic form of the SG Binder parameter g_{SG} in the thermodynamic limit, as expected in the present model.

the extrema are still faint and visible only for larger lattices, making a reliable estimate of T_{CG} and T_{SG} difficult. The inflection point would be located somewhere between T_{SG} and T_{CG} in the thermodynamic limit, and thus, can be utilized to give a lower bound of T_{CG} and an upper bound of T_{SG} . Indeed, the inflection point of the $L \geq 20$ data lie around $T \sim 0.31$, which is compatible with our present estimates of $T_{SG} = 0.274^{+0.012}_{-0.032}$ and $T_{CG} = 0.308 \pm 0.005$.

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